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Numerical Simulation of Heat Transfer in a Vertical Inverted U–Tube Steam Generator

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Abstract

In this paper, a numerical scheme has been developed for primary and secondary fluids adopting Euler–Cauchy method to find the temperature distribution of the primary fluid relative to the secondary fluid and vice–versa and to obtain an optimum boiling length of the tube in the vertical inverted U–tube steam generator (UTSG).

Key words: Steam generator; Temperature distributions; Numerical scheme; Optimum boiling length.

1. Introduction

Numerical simulation is infact, experimentation on the model rather than the system itself. In complex systems, numerical simulation is the only way in which variables affecting the design, their interaction with each other can be controlled and studied [1]. U–tube steam generator (UTSG) power plant has become better choice to produce energy, because there is a significant evolution of the water/steam flow pattern type due to the complete vaporization inside the tubes [2]. In addition to this non–uniform flows in multiple U–tubes during natural circulation tests in ROSA–IV large scale test facility were observed [3]. Srohmayer [4] has developed a dynamic model of vertical U–tube steam generators for operational safety systems.

To analyze the variations of temperatures in the primary and secondary sides of a UTSG, a mathematical model has been developed. The solution of the model has been discussed analytically which gives the temperature distribution of the primary fluid along the whole length of the tube. The change in the outlet temperature using change in the mass flow rate (m) and the factor $R * h$ is also computed. A graphical method is proposed to find the percentage changes in the outlet temperature of the primary fluid [5].

In the present work, it is noted that the analytical solution is based on certain simplifying assumptions, which to some extent limits the scope of its applicability. A numerical analysis scheme has been developed adopting Euler–Cauchy method to obtain a solution independent of the assumptions made in the analytical solution. This makes the model more realistic and flexible. The main advantage of the proposed method is that it gives a way to address upon the optimum boiling length of the tube along with the relative temperature distributions of primary and secondary fluids. A boundary condition needs to be prescribed along the tube to start calculations.

2. Mathematical Model

2.1 Governing Equations

The following equations describe the heat transfer in the control volume under transient and steady state conditions respectively.

$$\rho \pi R^2 Cp \frac{\partial T}{\partial t} = 2 \pi Rh T_f(x) - 2 \pi Rh T(x, t) - \rho \dot{v} Cp \frac{\partial T}{\partial x} \quad (1)$$

$$\rho \dot{v} Cp \frac{dT}{dx} + 2 \pi Rh T(x) - 2 \pi Rh T_f(x) = 0 \quad (2)$$

or

$$\frac{dT}{dx} + P T(x) = P T_f(x) \quad (3)$$

where $P = 2 \pi Rh / \rho \dot{v} Cp$

2.2 Primary and Secondary Loops

Primary loop transfers heat to secondary loop without phase change. Secondary fluid absorbs heat until it reaches its saturation temperature. Let L be the total length of the tube and L_b is the length at which secondary fluid attains saturation temperature as shown in figure 1.

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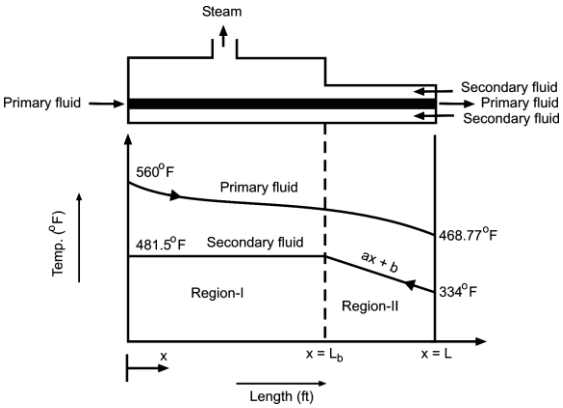


Figure 1. Temperature variations in primary and secondary fluids.

3. Numerical Models

3.1 Secondary Fluid

The following equation describes the numerical model for the secondary fluid.

$$\frac{d\,T}{d\,x} + Q\,T_s = Q\,T_p\,;$$

where $Q = 2\,\pi\,Rh / \rho\,\dot{v}\,Cp$ (4)

or $\frac{d\,T}{d\,x} = f\,(x, T)\,;\; f\,(x, T) = Q\,(T_p - T_s)$

which on adopting Euler–Cauchy scheme, takes the following form.

$$T_{s_{n+1}} = T_{s_n} + g\,f\,(x_n, T_n); \qquad (5)$$

where $f\,(x_n, T_n) = Q\,(T_{p_n} - T_{s_n})$;

$n = 0, 1, 2, 3, \ldots\ldots$

where g is the incremented step size which is uniform.

Here the parameters are employed for the secondary fluid.

3.2 Primary Fluid

The following equation describes the numerical model for the primary fluid

$$\frac{d\,T}{d\,x} + P\,T_p = P\,T_s\,;\; P = 2\,\pi\,Rh / \rho\,\dot{v}\,Cp \quad (6)$$

or $\frac{d\,T}{d\,x} = f\,(x, T);\; f\,(x, T) = P\,(T_s - T_p)$

which on adopting Euler – Cauchy Scheme takes the following form.

$$T_{p_{n+1}} = T_{p_n} + g\,f\,(x_n, T_n); \qquad (7)$$

where $f\,(x_n, T_n) = P\,(T_{s_n} - T_{p_n})$;

$n = 0, 1, 2, 3, \ldots..$

Here the parameters are employed for the primary fluid.

4. Model Layout and Solution Procedure

As shown in figure 2, the model data is read. Region – I and II describe how primary loop transfers heat to secondary loop. The admissible recurring constants are calculated according to the recorded data. First of all temperature distribution Ts is estimated in region – II with the assumed boiling length L_b.

Then with the given end and initial conditions the temperature distribution Tp is calculated using Ts and vice–versa from equations (7) and (5) respectively obtaining Lb_{new} at each time. The values calculated are relaxed with respect to that of previous iteration and the previous iteration values are then replaced by the resultant values. The procedure is repeated till the convergence criteria is met.

5. Implementation of Numerical Scheme

The iterative procedure must be started assigning initial values to the variables. The convergence depends upon the skill in the selection of values that are not too far from the real values. In the design point of view, some of the initial values would be the nominal whereas others would need to be guessed.

Therefore, the solution using the above numerical scheme is initiated by providing temperature distribution of the secondary fluid and an expected boiling length (L_b) as input to the computer program [5,6]. This program evaluates the temperature distribution of the primary fluid using the temperature distribution of secondary fluid and then evaluates the temperature distribution of secondary fluid using the temperature distribution of primary fluid and an iterative procedure is followed until the program converges for an optimum boiling length (L_b) using relative error criteria as shown in figure 3 (a) that is when all the absolute values of $(L_{b_{new}} - L_{b_{old}})/L_{b_{new}}$ are small enough the process is finished.

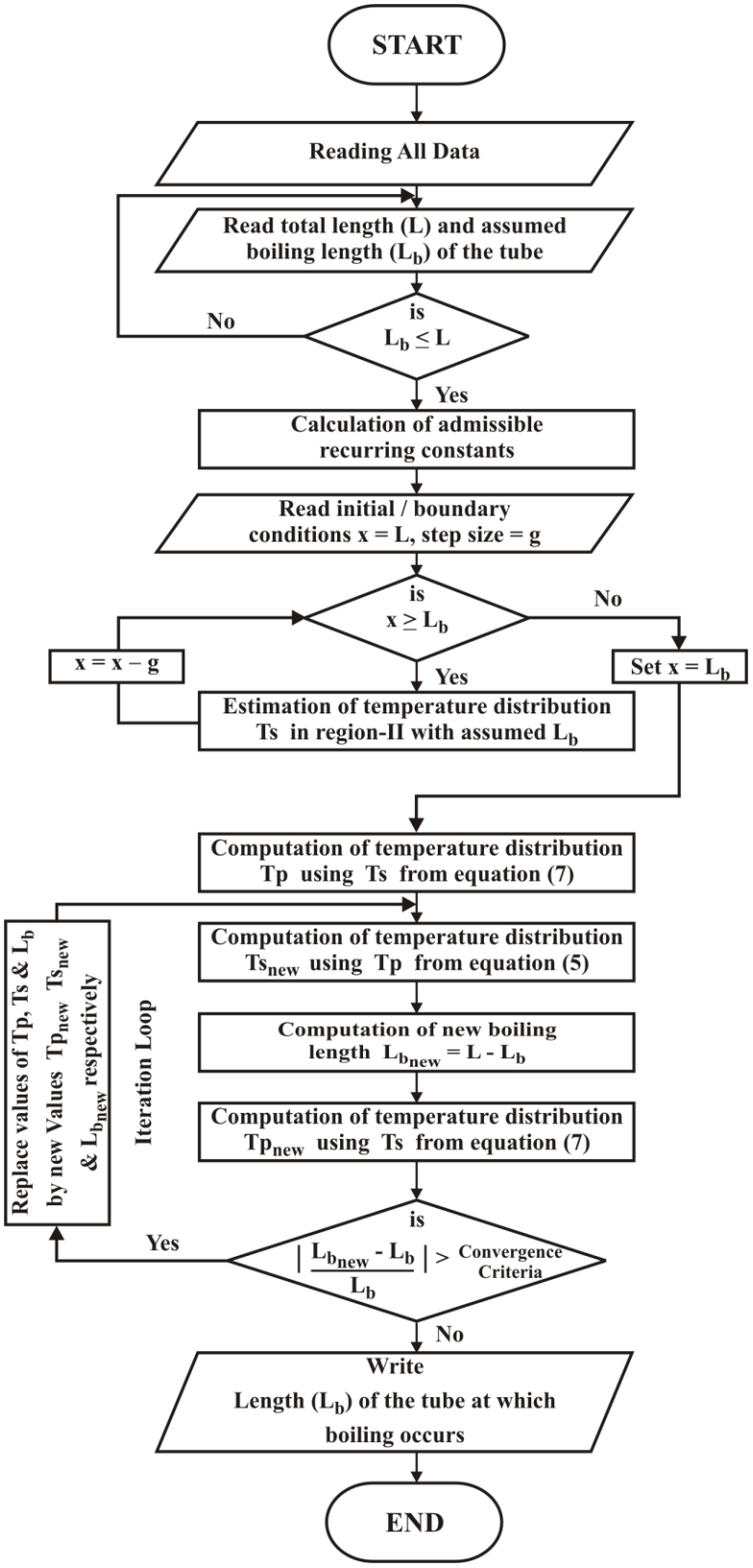


Figure 2. Flow chart of the steady state flow numerical model

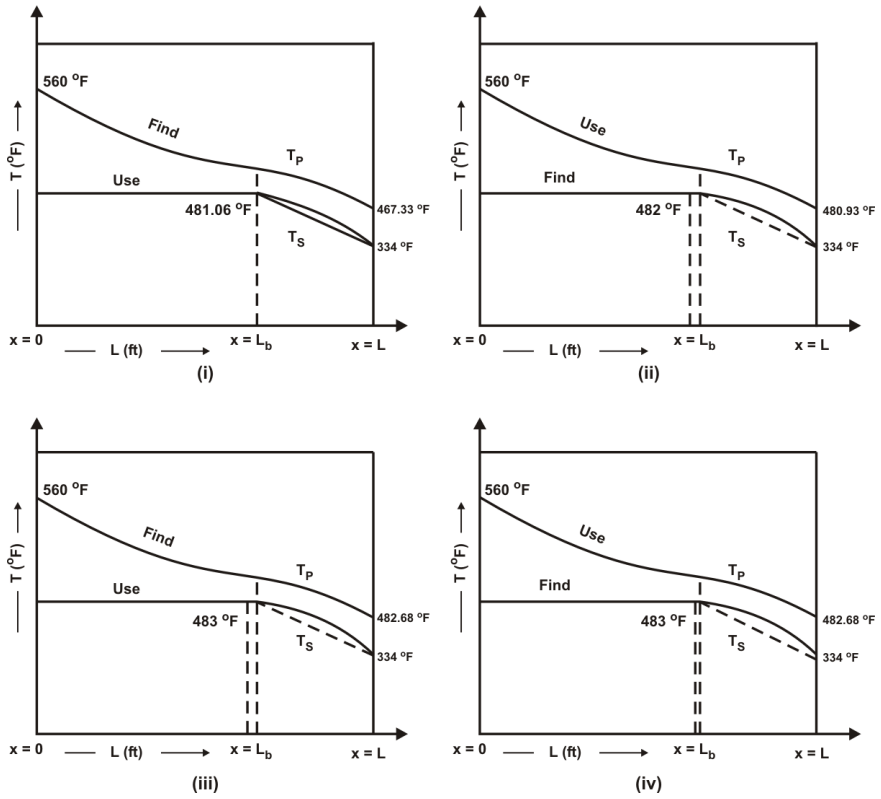


Figure 3(a): Temperature distribution of the secondary and primary fluids in a boiler loop.

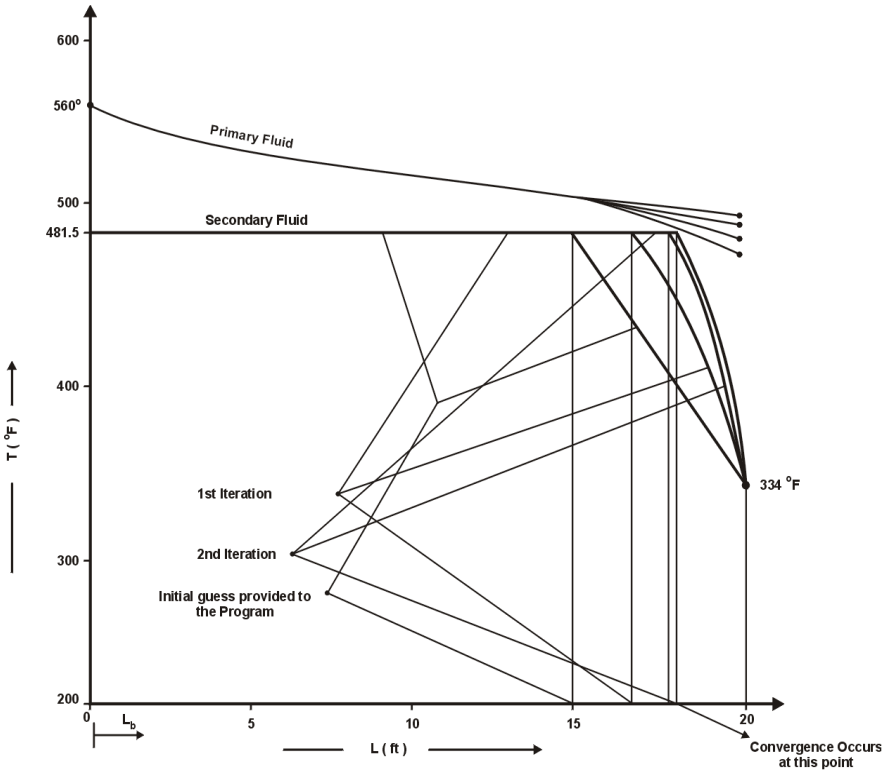


Figure 3(b): Temperature distribution curves for both types of fluids.

Furthermore, the program simultaneously gives the temperature distribution of secondary and primary fluids at each iteration along with the optimum boiling length L_b. Using the computed data, temperature distribution curves for both types of fluids are shown below in figure 3(b). The convergence is reached in only two steps because the initial values were very close to the solution. This scheme also provides the user freedom and flexibility in the selection of input data.

6. Results and Discussion

For different values of the product $R * h$ (10,000 to 100,000) and different lengths of the tubes (10 ft to 60 ft), T_{pout} is noted as shown in table 1.

Table 1. Computed results for different values of $R * h$ & tube length (L).

R*h	L = 10 (T_{pout})	L = 20 (T_{pout})	L = 30 (T_{pout})	L = 40 (T_{pout})	L = 50 (T_{pout})	L = 60 (T_{pout})
10,000	535.35	519.94	508.47	499.32	492.43	486.75
20,000	519.28	594.93	486.33	478.23	473.36	470.09
30,000	507.26	485.97	475.30	469.85	467.23	465.30
40,000	493.47	477.51	469.62	466.47	465.22	464.73
50,000	489.92	472.51	466.87	465.08	464.52	464.35
60,000	484.62	469.16	465.44	464.48	464.27	464.21
70,000	480.00	467.01	464.61	464.18	464.08	464.10
80,000	476.08	464.60	464.27	464.01	464.00	463.90
90,000	473.04	464.98	463.98	463.87	463.87	463.00
100,000	470.54	464.36	463.76	426.18	415.99	407.07

Graphs of $R * h$ vs T_{pout} for different lengths of the tube are plotted as shown in figure 4. If T_{pout} (for a particular tube length L) and h are known, then these graphs can be used to evaluate: R, the number of tubes in the bundle.

For different values of the product $R * h$ at a particular length L with other parameters remaining unchanged, we observe that as the value of this product increases, then T_{pout} decreases, that is more heat will be transferred to the secondary fluid and more steam will be generated.

For specified values of the product $R * h$, if the tube length L is increased, a similar effect is observed on T_{pout}, that is, with the increase of length, T_{pout} decreases and as a result more heat will be transferred to the secondary fluid which also increases the capacity of steam.

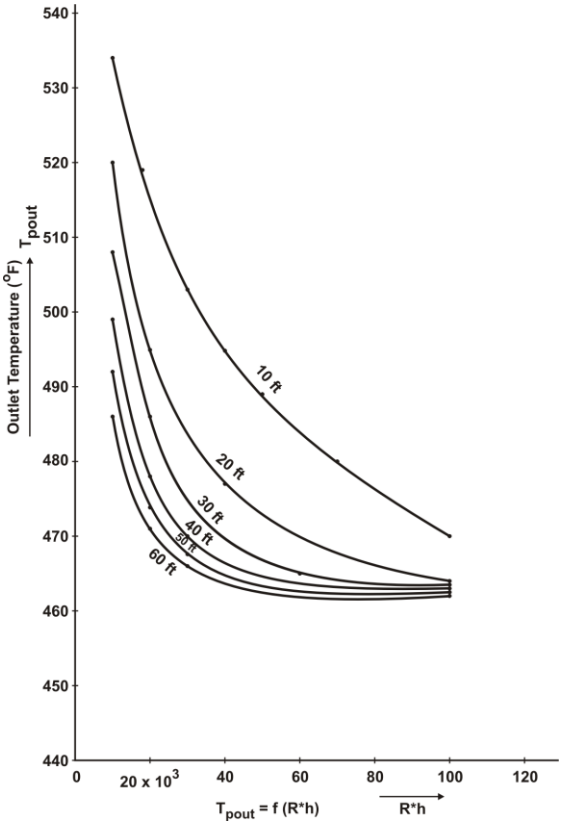


Figure 4. Optimum tube length using T_{pout} and $R * h$

7. An Example

If $R * h$ is given to be 70 (Btu/hr – ft ^oF) and T_{pout} is 480 ^oF then to decide what should be the tube length for achieving an optimum heat transfer rate? The graphs shown in figure 4 can be used to obtain the answer. From these graphs for a particular data mentioned above, the optimum tube length is obtained at the intersection of the values of T_{pout} and the factor $R * h$. It is found that the length of the tube is 10 ft. Similarly in most of the boiler design problems, to find the value of heat transfer coefficient h by the conventional methods is difficult and time consuming task. But the graphs plotted in figure 4 provide a convenience to the designer, as he can use these graphs and with T_{pout}, tube length, number of tubes and radius of tube, he can determine the value of heat transfer coefficient easily. This method seems to be suitable to define operating rules connected to measurable parameters.

8. Conclusion

The optimum boiling length of the tube (L_b) has been obtained using relative error criteria applied in

the numerical scheme. A graphical method with an example is also given to determine the value of heat transfer coefficient easily. Furthermore, the numerical results obtained from the model and experiment are in good agreement.

References

[1] Marie – Nöelle, D., Georges, H; "Computer and Chemical Engineering 28(2004) 651–660.

[2] Jeong, J. J., Hwang, M., Lee, Y. J; Int. Comm. Heat Mass Transfer, 30(2003) 643-652.

[3] Kukita, Y. M., Nakamura, H., Tasaka, K;, Nucl. Sci. Eng. 99(1988) 289-298.

[4] Strohmayer, W. H; Dynamic Modeling of Vertical U-tube Steam Generators for Operational Safety Systems. Ph.D. Thesis. The Department of Nuclear Engineering, Massachusetts Institute of Technology, (1984), 4.19-4.25.

[5] Shahid, N. A., Ibrahim, K. M; Simulation of Steam Generator in a Power Plant, a report submitted to CTC, UGC, Islamabad, (1988).

[6] Barbara Johnston, "C++ Programming Today", Prentice–Hall of India Private Limited, New Delhi – 110001, 2002.

Nomenclature

h	convective heat transfer coefficient (Btu/hr – ft^2 ^oF)
ρ	density of fluid (Lb/ft^3)
Cp	heat capacity at constant pressure (Btu/Lb ^oF)
L	length of the tube (ft)
L_b	length of the tube where boiling occurs (ft)
$\dot{v}$	volume flow rate of the secondary fluid (ft^3/hr)
R	equivalent radius of tubes (inch)
m	mass flow rate of the primary fluid (Lbs/hr)
T_{sat}	saturation temperature of the secondary fluid (^oF)
$T_{f(x)}$	temperature of the fluid on the surface of the tube due to convection (^oF)
T_s	temperature of the secondary fluid
T_p	temperature of the primary fluid
S_n	secondary cycle
P_n	primary cycle