Pak. J. Engg. Appl. Sci. Vol. 33 July, 2025 (p. 83–87)

Enhancing Terrestrial Water Storage Understanding Using Combination of Finite Difference Numerical Method and Remote Sensing Data Integration

Faraz Ul Haq[1,2 *], Muhammad Fahim Aslam[1,2]

1. *Department of Civil Engineering, University of Memphis, 3815 Central Ave, Memphis, TN, 38152*
2. *Center for Applied Earth Science and Engineering Research, The University of Memphis, 1000 Wilder Tower, 3675 Alumni Ave., Memphis, TN 38152, USA*

* **Corresponding Author**: Email: fuhaq@memphis.edu

Abstract

Terrestrial Water Storage (TWS) plays a vital role in the hydrological cycle, significantly impacting water resource management, agriculture, and climate research. Reliable estimation of TWS is crucial for forecasting droughts, optimizing reservoir management, and assessing the effects of climate change on water resources. However, traditional ground-based methods for estimating TWS are often limited in spatial coverage and are resource-intensive. This article aimed to numerically model these TWS changes through integration of remote sensing data and finite difference method. Finite Difference method. The proposed method utilized the soil moisture, evapotranspiration, vegetation and precipitation data to simulate the changes in water storage over time for the Shelby County. The scripts were run in Google Earth Engine (GEE) and MATLAB environment for remote sensing data analysis and simulation, respectively. Evapotranspiration was calculated as a nonlinear function of temperature and Normalized Difference Vegetation Index (NDVI) using the Newton-Raphson method. The results indicated the increase in TWS anomalies which showed a net gain in water storage. The continuous rise in TWS suggests increased water retention, which could be attributed to higher precipitation, reduced evapotranspiration (ET), or changes in soil moisture and groundwater levels. The outcomes of this article will be helpful to identify the water storage changes which are crucial for water resources management.

Keywords: Terrestrial Water Storage; Finite Difference Method; Newton Raphson Method; Water Resources Management

1. Introduction

Groundwater is indeed an important source of fresh water for agricultural, domestic, and industrial needs worldwide [1], [2]. Several aquifers, including Central Valley, Indus Plain, Middle East, and others, are facing several anthropogenic stresses caused by population growth, industrialization, and urbanization [3], [4], [5], [6]. Groundwater systems are inherently dynamic, responding to external stresses such as pumping, precipitation, irrigation, surface flow fluctuations, and evapotranspiration. It has long been recognized that understanding and predicting the responses of groundwater systems is crucial for sustainable water management, ensuring reliable drinking water supplies, and supporting economic and industrial developments. Thus, various approaches have been employed over the past decades by hydrogeologists. These traditionally included mathematical and numerical models and field observation networks. Mathematical models aimed to simulate groundwater flow and its interactions with hydrological components such as rivers and lakes, and, more recently, advanced artificial intelligence techniques that leverage large datasets for improved predictive accuracy [7], [8]. Observation networks have also been used for many years to monitor groundwater level fluctuations. These networks consist of strategically placed wells that measure water levels over time. However, developing and data collection processes from the monitoring wells are costly and, in many cases, limited to smaller areas, which hinders the capture of spatially dynamic groundwater behavior. However, this limitation is being addressed with the usage of advanced earth gravity satellites, i.e., GRACE (Gravity Recovery and Climate Experiment) mission.

GRACE satellite mission is one of the remote sensing tools developed by NASA and the German Aerospace Center, launched in 2002, which aims to map Earth's gravity field variations with high precision [9]. The continuity of the mission was covered by the GRACE Follow-On mission (GRACE-FO) launched in 2018. By observing changes in the Earth's gravity field, changes in the amount of water stored in a region, generally referred to as terrestrial water storage (TWS), can be estimated. TWS includes all forms of water on and under the ground, such as groundwater, snow water, surface water, soil moisture storage, ice, and biomass water. Another approach to estimate TWS is through numerical modelling which is based on water balance equation. Various numerical modelling method are available which includes finite difference, finite volume, finite element and Monte Carlo simulation method. All these numerical methods provide an approximate solution of differential equations in time or space. TWS water balance equation accounts for the rate of change in TWS over time as a function of input parameters and the losses i.e., runoff and evapotranspiration. This is shown in the equation 1 [10].

$$\frac{d}{dt}(TWS) = P - ET - R + \Delta SM + \Delta GW \quad \text{Eq. (1)}$$

where, P is the precipitation, ET is the evapotranspiration, R is the runoff, SM is the soil moisture, and GW is the groundwater. It can be derived from Eq. 1, that GW can be obtained by removing the contributions of other components (i.e., P, ET, R *and* SM) from TWS. The components mentioned in equation 1 can be assessed using specialized satellite data, detailed hydrological models, and direct ground-based measurements. For example, soil moisture can be obtained from Soil Moisture Active Passive (SMAP) satellite mission or by direct measurement using gravimetric method. Similarly, groundwater can be obtained using monitoring wells, pumping tests and Electrical Resistivity Tomography (ERT). Also, surface water can be measured using current meter, and other remote sensing tools including MODIS (Moderate Resolution Imaging Spectroradiometer). However, if these data sets are not available publically due to lack of in situ observations during the period of interest, the monthly output from global, gridded hydrological land surface models (LSMs) can be used to separate groundwater storage from other components mentioned in equation 1. This article aimed to numerically model the TWS changes using finite difference method. The objectives of the work are

- Integration of multiple remote sensing data sources (precipitation, soil moisture, groundwater levels, temperature, and NDVI) into a comprehensive TWS model.
- To solve the TWS equation and track water storage changes across spatial nodes.

2. Study Area

Shelby county was selected as the study area for this study with spatial coordinates of 90.31074524°W, 89.63301086°W, 34.99597549°N, 35.40962601°N The primary source of drinking water in the Shelby County is the groundwater. The water comes from the Memphis aquifer which is also known as Sparta aquifer. Some of the wells also pumps water from the deeper fort pillow aquifer. Shelby County is endowed with three primary freshwater aquifers: the shallow aquifer, the Memphis aquifer, and the Fort Pillow aquifer. The shallow aquifer, with a thickness ranging from 0 to 30 meters, comprises alluvial and fluvial deposits extending across the entire county. This aquifer includes the Mississippi River Valley Alluvial (MRVA) aquifer on the western side of the bluff line. In the eastern part of the county, the shallow aquifer aligns with the unconfined region of the Memphis aquifer, serving as a crucial recharge zone assessment of the potential for contamination of the Memphis aquifer in the Memphis area. Below the shallow aquifer lies the upper Claiborne confining unit (UCCU), acting as a regional confining unit for the Memphis aquifer. The cross section of the Memphis aquifer is shown in the Figure 1.

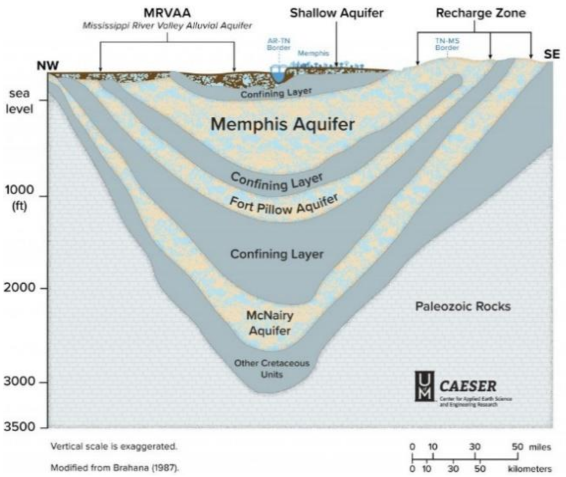

Fig. 1: Cross section of the Memphis aquifer (Source: CAESER)

3. Material and Methodology

The methodology involved the gridding of the study are into equal grid size od 10km x 10km. Google Earth Engine (GEE) code was prepared to obtain the soil moisture, temperature, NDVI and precipitation datasets for each of these grids for a period between January 2020-October 2023. After that, Newton Raphson was applied to obtain the evapotranspiration as a nonlinear function of NDVI and temperature. Finite difference method was applied to numerically model the TWS changes based on MATLAB coding. At the end, raw and smooth TWS curves were obtained. The flow chart of the methodology is shown in Figure 2.

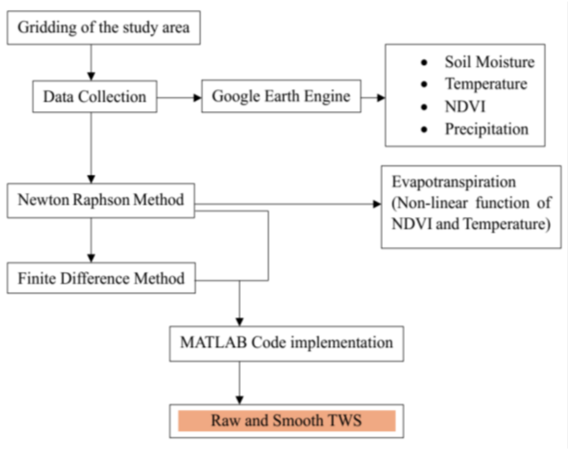

Fig. 2: Flow chart of the steps involved in the methodology

3.1 Data Collection

Different remote sensing platforms were used to collect the data for the numerical modelling. Precipitation data was obtained from IMERG (Integrated Multi- satellite Retrievals for GPM), temperature, soil moisture and NDVI was obtained from MODIS (Moderate Resolution Imaging Spectroradiometer). The google earth engine code was developed to obtain the required data for modelling. The study area was divided into grids of 10km x 10km and the data was downloaded for each grid. These datasets were integration into the numerical model for the estimation of TWS changes overtime. However, the validation of these remote sensing datasets were not performed since it was out of the scope of the study. The gridded map of the study area is shown in the Figure 3.

Fig. 3: Gridded map of the study area

3.2 Newton Raphson method for Evapotranspiration Calculation

Evapotranspiration was calculated as a nonlinear function of temperature and NDVI using the Newton Raphson method as adopted [11] for the estimation of actual evapotranspiration. The iteration process refined the ET values by accounting the effects of temperature and NDVI. The scaling factor (k) of 0.002 was used for NDVI. The non linear function for the estimation of ET is shown in the equation 2.

$$\text{Evapotranspiration} = f(x) = ET^2 - (ndvi * temp * k) \qquad \text{Eq. (2)}$$

3.3 Finite difference method

The Finite Difference Method was adopted to calculate TWS at each node over time by integrating precipitation, ET, soil moisture, and groundwater changes. It is a numerical technique to approximate the solutions of differential equations. The FDM divides the continuous domain into discrete points e.g., time interval t was divided into t0, t1,t2, The explicit method was used for numerical modeling which calculates the value at next time step by using known values from the current time step. This method was adopted since it is easy to implement and can be efficient for small problems. The temporal resolution for the FDM was dt=0.01 which represents the discrete time increment in the model while the model contains 100 spatial nodes. The model is based on the water balance equation, which accounts for precipitation, evapotranspiration (ET), soil moisture change, and groundwater level changes as shown in the equation 3.

$$TWS(i, t+1) = TWS(i, t) + dt.(P(t) - ET(t) + \Delta SM(t) + \Delta GW(t)) \qquad \text{Eq. (3)}$$

3.4 MATLAB Code Implementation

MATLAB code was developed to estimate and numerically model the TWS using finite difference method. A tolerance of $1\times10{-}6$was fixed for checking the convergence and 100 iterations were performed for Newton Raphson method. Model parameters were defined to implement the finite difference method which are given below:

dt = 0.01; #Length of each time step

T = 100 #Total number of time steps

N = 100; #Number of spatial nodes

k_ndvi = 0.002; #Scaling factor

The steps which were followed to develop the code are:

- Importing the Excel file
- Extract Columns from data
- Initializing arrays and time stepping loop
- ET using Newton Raphson method
- Update TWS for spatial nodes
- Storing the updated TWS values
- Plotting the TWS distribution

4. Results and discussions

The model produces both Raw TWS and Smooth TWS for analysis. Raw TWS reflects the direct outputs from each timestep in the simulation, showing point-by-point changes in storage (Figure 4). Smooth TWS, generated using a moving average, highlights seasonal and long-term trends by reducing short-term variability. The results are shown in the Figure 3. The final graph includes Raw TWS which is the direct model output, showing detailed timestep changes in TWS and smooth TWS which showed the smoothed trend line, revealing broader seasonal and annual patterns. The results indicated a general increase in TWS anomalies over the period (2020-2023) which showed the gain in water storage. The possible caused for this gain in water storage can be increase in the precipitation and vegetation. The increase in water storage was more prominent during the mid-2020 and early 2023. However, during the mid-2022, a slower increase was observed which suggested a possible balance between recharge and depletion. There are also periodic fluctuations which likely correspond to seasonal hydrological cycle i.e., wet and dry seasons. Also, smoothening of TWS data played an important role to reveal the long-term hydrological pattern by removing the short-term fluctuations. No negative TWS anomalies (drought events) are visible suggesting a relatively stable or wet hydrological conditions during the study period.

5. Conclusions and Recommendations

The results of the MATLAB simulation indicate a steady increase in Terrestrial Water Storage (TWS) anomalies from January 2020 to October 2023. The raw data (solid blue line) exhibits short-term fluctuations, while the smoothed data (dashed orange line) provides a clearer picture of the overall trend. The continuous rise in TWS suggests increased water retention, which could be attributed to higher precipitation, reduced evapotranspiration (ET), or changes in soil moisture and groundwater levels. There are minor dips, such as in late 2021, which may correspond to seasonal variations or climate-related factors. Since ET is estimated using the Newton-Raphson method based on temperature and NDVI, its impact on TWS must be further explored. To gain deeper insights, it would be useful to compare these results with precipitation and temperature patterns, analyze spatial variations, and conduct sensitivity analysis by adjusting model parameters. Overall, the study demonstrates a positive water storage trend, highlighting the importance of further analysis to confirm the driving factors behind these changes.

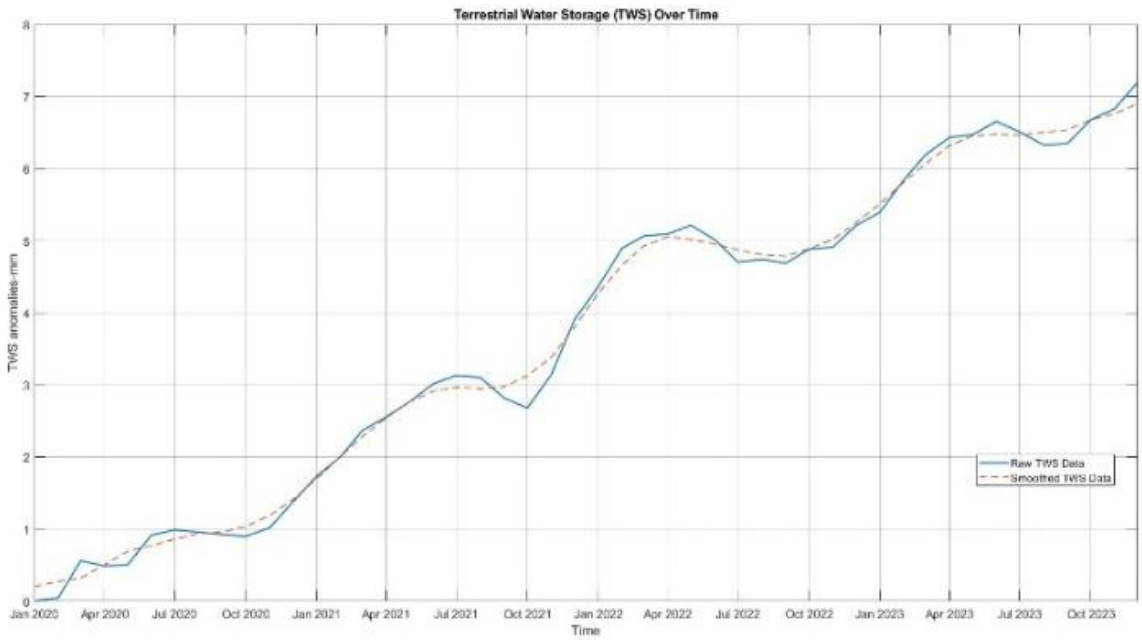

Fig. 4: TWS changes over time

It is recommended to compare the TWS data obtained from numerical modelling with the GRACE satellite TWS data to ensure the validity of TWS.

6. References

[1] Döll, P., & Siebert, S. (2002). Global modeling of irrigation water requirements. *Water Resources Research, 38*(4), 8-1–8-10. https://doi.org/10.1029/2001wr000355

[2] Wada, Y., et al. (2016). Modeling global water use for the 21st century: The Water Futures and Solutions (WFaS) initiative and its approaches. *Geoscientific Model Development, 9*(1), 175–222. https://doi.org/10.5194/gmd-9-175-2016

[3] Giglou, A. N., Nazari, R. R., Jazaei, F., & Karimi, M. (2023). Numerical analysis of surface hydrogeological water budget to estimate unconfined aquifers recharge. *Journal of Environmental Management, 346*, 118892. https://doi.org/10.1016/j.jenvman.2023.118892

[4] Giglou, A. N., Nazari, R., Jazaei, F., & Karimi, M. (2023). Assessing the effects of increased impervious surface on the aquifer recharge through river flow network, case study of Jackson, Tennessee, USA. *Science of the Total Environment, 872*, 162203. https://doi.org/10.1016/j.scitotenv.2023.162203

[5] Jazaei, F., Simpson, M. J., & Clement, T. P. (2016). Spatial analysis of aquifer response times for radial flow processes: Nondimensional analysis and laboratory-scale tests. *Journal of Hydrology, 532*, 1–8. https://doi.org/10.1016/j.jhydrol.2015.11.024

[6] Barlow, P. (n.d.). *Ground Water in Freshwater-Saltwater Environments of the Atlantic Coast*. U.S. Geological Survey Circular 1262.

[7] Azizi Mobaser, J., Rasoulzade, A., & Abedi, A. (2021). Modification of DRASTIC Model with Two Different Methods for Assessing Groundwater Vulnerability in Ardabil Plain. *Journal of Irrigation Science and Engineering, 44*(2), 129–146. https://doi.org/10.22055/JISE.2021.36627.1963

[8] Perry, R. (2001). Hydrogeological investigations and numerical simulation of groundwater flow in the karstic aquifer of northwestern Yucatan, Mexico. In *First International Conference on Saltwater Intrusion and Coastal Aquifers*, Morocco.

[9] Frappart, F., & Ramillien, G. (2018). Monitoring groundwater storage changes using the Gravity Recovery and Climate Experiment (GRACE) satellite mission: A review. *Remote Sensing, 10*(6), 829. https://doi.org/10.3390/rs10060829

[10] Humphrey, V., Rodell, M., & Eicker, A. (2023). Using satellite-based terrestrial water storage data: A review. *Surveys in Geophysics, 44*(5), 1489–1517. https://doi.org/10.1007/s10712-022-09754-9

[11] Taheri, M., Mohammadian, A., Ganji, F., Bigdeli, M., & Nasseri, M. (2022). Energy-based approaches in estimating actual evapotranspiration focusing on land surface temperature: A review of methods, concepts, and challenges. *Energies, 15*(4), 1264. https://doi.org/10.3390/en15041264